1
Interference

1.1 INTRODUCTION

In this chapter we will discuss the phenomena associated with the interference of light waves. At any point where two or more wave trains cross one another they are said to interfere. In studying the effects of interference we are interested to know the physical effects of superimposing two or more wave trains.

It is found that the resultant amplitude and consequently, the intensity of light gets modified when two light beams interfere. This modification of intensity obtained by the superposition of two or more beams of light is called interference. In order to find out resultant amplitude, when two waves interfere, we make use of the principle of superposition. The truth of the principle of superposition is based on the fact that after the waves have passed out of the region of crossing, they appear to have been entirely uninfluenced by the other set of waves. Amplitude, frequency and all other characteristics of each wave are just as if they had crossed an undisturbed space. *The principle of superposition states that the resultant displacement at any point and at any instant may be found by adding the instantaneous displacements that would be produced at the point by the individual wave trains if each were present alone.* In the case of light wave, by displacement we mean the magnitude of electric field or magnetic field intensity.

1.2 SUPERPOSITION OF WAVES

1.2.1 Superposition of Waves of Equal Phase and Frequency

Let us assume that two sinusoidal waves of the same frequency are travelling together in a medium. The waves have the same phase, without any phase angle difference between them. Then the crest of one wave falls exactly on the crest of the other wave and so do the troughs. The resultant amplitude is got by adding the amplitudes of each wave point by point. The resultant amplitude is the sum of the individual amplitudes (Fig. 1.1).

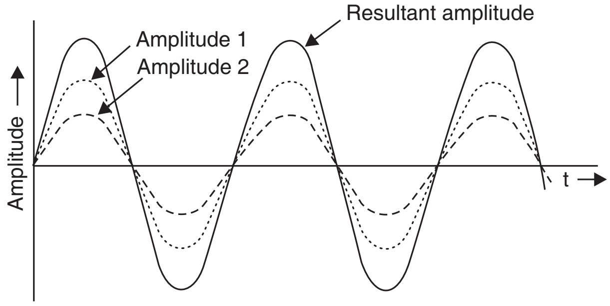

Fig. 1.1 Superposition of waves of equal phase and frequency

i.e., $A = A_2 + A_2 + \ldots\ldots$

The resultant intensity is the square of the sum of the amplitudes

$$I = (A_1 + A_2 + A_3 + \ldots\ldots)^2 \tag{1.1}$$

1.2.2 Superposition of Waves of Constant Phase Difference

Let us consider two waves that have the same frequency but have a certain constant phase angle difference between them. The two waves have a certain differential phase angle ϕ. In this case the crest of one wave does not exactly coincide with the crest of the other wave (Fig. 1.2). The resultant amplitude and intensity can be obtained by trigonometry.

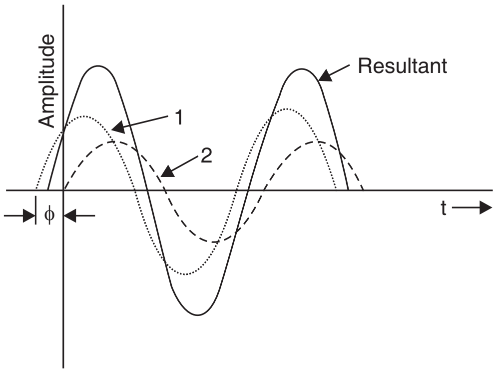

Fig. 1.2 Superposition of two sine waves of constant phase difference

The two waves having the same frequency ($\omega = 2\pi f$) and a constant phase difference (ϕ) can be represented by the equations

$$Y_1 = a \sin \omega t$$
$$Y_2 = b \sin (\omega t + \phi) \tag{1.2}$$

where ϕ is the constant phase difference, a, b are the amplitudes and ω is the angular frequency of the waves. The resultant amplitude Y is given by

$$\begin{aligned} Y &= Y_1 + Y_2 \\ &= a \sin \omega t + b \sin (\omega t + \phi) \\ &= a \sin \omega t + b (\sin \omega t \cos \phi + \cos \omega t \sin \phi) \\ &= a \sin \omega t + b \sin \omega t \cos \phi + b \cos \omega t \sin \phi \\ &= (a + b \cos \phi) \sin \omega t + b \cos \omega t \sin \phi \end{aligned} \tag{1.3}$$

If R is the amplitude of the resultant wave and θ is the phase angle then

$$\begin{aligned} Y &= R \sin (\omega t + \theta) \\ &= R \{\sin \omega t \cos \theta + \cos \omega t \sin \theta\} \\ &= R \cos \theta \sin \omega t + R \sin \theta \cos \omega t \end{aligned} \tag{1.4}$$

Comparing Eqs. (1.3) and (1.4)

$$R \cos \theta = a + b \cos \phi$$
$$R \sin \theta = b \sin \phi$$
$$\Rightarrow \quad R^2 = a^2 + b^2 + 2ab \cos \phi$$
$$\theta = \tan^{-1} \frac{b \sin \phi}{a + b \sin \phi} \tag{1.5}$$

Clearly, R is maximum when $\phi = 2n\pi$

and is minimum when $\phi = (2n + 1)\pi$ where $n = 0, 1, 2, 3, ...$

When ϕ is an even multiple of π we say that waves are in phase and when ϕ is an odd multiple of π, the waves are out of phase.

When the amplitude of waves are equal to a say, then

$$I = 2a^2 (1 + \cos \phi) = 4a^2 \cos^2 \phi/2 \tag{1.6}$$

A plot of I versus ϕ is shown in Fig. 1.3. Clearly , this reveals that the light distribution from the superposition of waves will consist of alternately bright and dark bands called interference fringes. Such fringes can be observed visually if projected on a screen or recorded photo-electrically. In the above discussion we have not considered travelling waves (*i.e.,* waves in which displacement is also a function of distance). If λ is the wavelength, then the change of phase that occurs over a distance λ is 2π. Thus, if the difference in phase between two waves arriving at a point is 2π, then difference in the path travelled by these waves is λ. Let the phase difference of two waves arriving at a point be δ and the corresponding path difference be x. For a path difference of λ, the phase difference = 2π. Therefore, for a path difference of x.

$$\text{Phase difference} = \delta = \frac{2\pi}{\lambda} . x = \frac{2\pi}{\lambda} . \text{ path difference}$$

and $$\text{Path difference} = x = \frac{\lambda}{2\pi} \text{ phase difference}$$

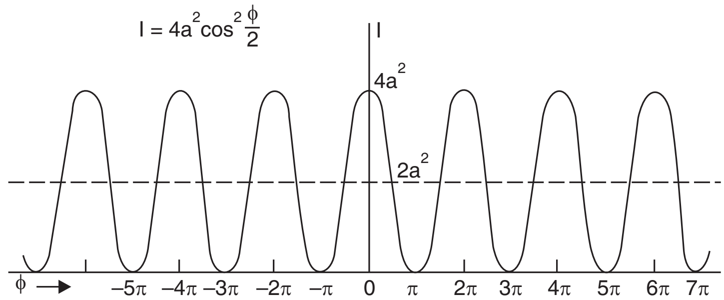

Fig. 1.3 Intensity distribution for the interference fringes from two waves of same frequency and amplitude

1.2.3 Superposition of Waves of Different Frequencies

So far we have assumed that the waves have the same frequency. But light is never truly monochromatic. Many light sources emit quasimonochromatic light *i.e.*, light emitted will be predominantly of one frequency but will still contain other ranges of frequencies. When waves of different freqencies are superimposed, the result is more complicated.

1.2.4 Superposition of Waves of Random Phase Differences

When waves having random phase differences between them superimpose, no discernible interference pattern is produced. The resultant intensity is got by adding the square of the individual amplitudes,

i.e., $$I = \sum_{i=1}^{N} A_i^2 = A_1^2 + A_2^2 + A_3^2 + \ldots\ldots \tag{1.7}$$

1.3 YOUNG'S DOUBLE SLIT EXPERIMENT

We have seen in the previous section that two waves with a constant phase difference will produce an interference pattern. Let us see how it can be realized in practice. Let us use two conventional light sources (like two sodium lamps) illuminating two pin holes (Fig. 1.4). Then we will find that no interference pattern is observed on the screen. This can be understood from the following reasoning. In a conventional light source, light comes from a large number of independent atoms each atom emitting light for about 10^{-9} seconds *i.e.,* light emitted by an atom, is essentially a pulse lasting for only 10^{-9} seconds. Even if the atoms were emitting under similar conditions, waves from different atoms would differ in their initial phases. Consequently light coming out from the holes S_1 and S_2 will have a fixed phase relationship for a period of about 10^{-9} sec. Hence, the interference pattern will keep on changing every billionth of a second. The human eye can notice intensity changes which last at least for a tenth of a second and hence we will observe a uniform intensity over the screen. However, if we have a camera whose time of shutter can be made less than 10^{-9} sec, then the film will record an interference pattern. *We can summarize the above argument by noting that light beams from two independent sources do not have a fixed phase relationship over a prolonged time period and hence, do not produce any stationary interference pattern.*

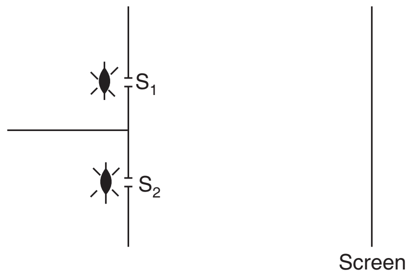

Fig. 1.4 If two sodium lamps illuminate two pin holes S_1 and S_2 no interference pattern is observed on the screen

Thomas Young in 1802 devised an ingenious but simple method to lock the phase relationship between two sources. The trick lies in the division of a wave front into two. These two split wave fronts act as if they emanated from two sources having a fixed phase relationship and therefore, when these two waves were allowed to interfere, a stationary interference pattern was produced. In the actual experiment a light source illuminated a tiny pin hole S (Fig. 1.5).

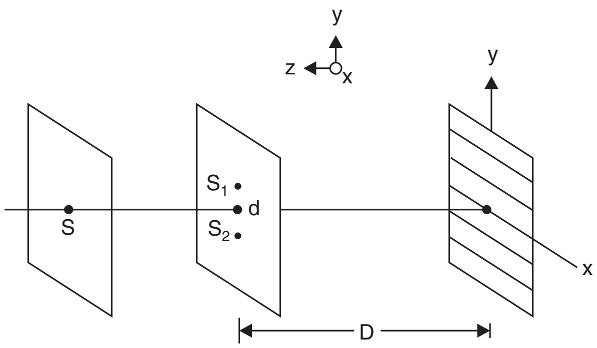

Fig. 1.5 Young's arrangement to produce interference pattern

Light diverging from this pin hole fell on a barrier containing two rectangular apertures S_1 and S_2 which were very close to each other and were located equidistant from S. Spherical waves travelling from S_1 and S_2 were coherent and on the screen beautiful interference fringes (Fig. 1.5) could be obtained. In the centre screen, where the light waves from two slits have travelled through equal distances and where the path difference is zero, we have zeroth-order maximum (Fig. 1.6). But maxima will also occur whenever the path difference is one wavelength λ or an integral multiple of wavelength $n\lambda$. The integer n is called the order of interference.

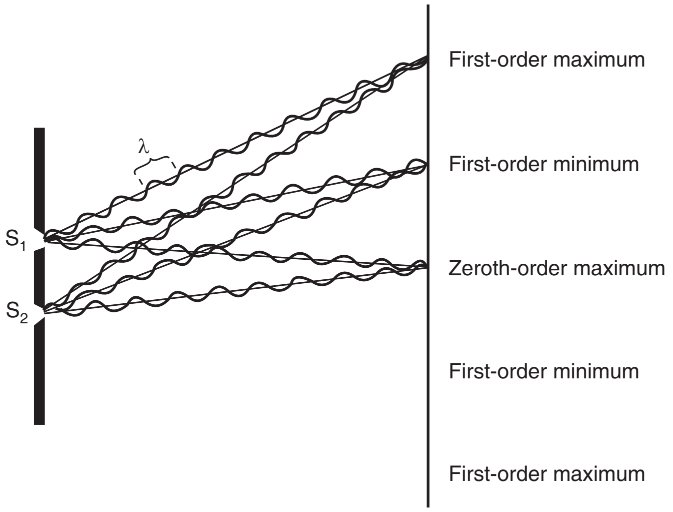

Fig. 1.6 Maxima and minima in Young's double slit experiment

When the path difference is a multiple of $(n + 1/2)\lambda$ we observe a dark fringe.

In order to calculate the position of the maxima, we proceed as follows. Let d be the distance between the slits and D be the distance of the screen from the slits.

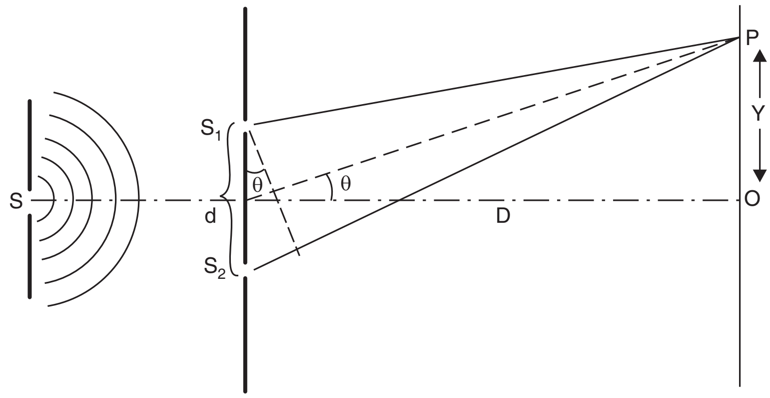

Fig. 1.7 Path difference in Young's double slit experiment

Let P be the position of the maximum (Fig. 1.7). Then the path difference between the two waves reaching P is

$$d \sin\theta = n\lambda \qquad \text{or} \qquad \sin\theta = \frac{n\lambda}{d} \quad (n = 1, 2, 3, ...)$$

where λ is the wavelength of light used and θ is the angle as shown in Fig. 1.7. If Y is the distance of point P from O, the centre of the screen, then we have

$$Y = D \tan\theta$$

For small angles of θ, $Y = D\tan\theta = D\sin\theta$

$$Y = \frac{Dn\lambda}{d} \quad \text{or} \quad \lambda = \frac{dY}{Dn} \tag{1.8}$$

Clearly, fringe width $= Y_{n+1} - Y_n = \beta = \dfrac{D\lambda}{d}$ (1.9)

Hence, by measuring the distance between slits, the distance to the screen and the distance from the central fringe to some fringe on either side, the wavelength of light producing the interference pattern may be determined.

1.4 COHERENCE

An important concept associated with the idea of interference is coherence. Coherence means that two or more electromagnetic waves are in a fixed and predictable phase relationship to each other. In general the phase between two electromagnetic waves can vary from point to point (in space) or change from instant to instant (in time). There are thus two independent concepts of coherence namely temporal coherence and spatial coherence.

Temporal Coherence **:** This type of coherence refers to the correlation between the field at a point and the field at the same point at a later time *i.e.* the relation between $E(x, y, z, t_1)$ and $E(x, y, z, t_2)$. If the phase difference between the two fields is constant during the period normally covered by observations, the wave is said to have temporal coherence. If the phase difference changes many times and in an irregular way during the shortest period of observation, the wave is said to be non coherent.

Spatial Coherence **:** The waves at different points in space are said to be space coherent if they preserve a constant phase difference over any time t. This is possible even when two beams are individually time incoherent, as long as any phase change in one of the beams is accompanied by a simultaneous equal phase change in the other beam (this is what happens in Young's double slit experiment). With the ordinary light sources, this is possible only if the two beams have been produced in the same part of the source.

Time coherene is a characteristic of a single beam of light whereas space coherence concerns the relationship between two separate beams of light. Interference is a manifestation of coherence.

Light waves come in the form of wave trains because light is produced during deexcitation of electrons in atoms. These wave trains are of finite length. Each wave train contains only a limited number of waves. The length of the wave train Δs is called the *coherence length*. It is the product of the number of waves N contained in wave train and their wavelength λ *i.e.*, $\Delta s = N\lambda$. Since velocity is defined as the distance travelled per unit of time, it takes a wave train of length Δs, a certain length of time 'Δt', to pass a given point

$$\Delta t = \Delta s / c$$

where c is the velocity of light. The length of time Δt is called the *coherence time*. The degree of temporal coherence can be measured using a Michelson's interferometer.

It is clear from the above discussion that the important condition for observing interference is that the two sources should be coherent. The observations of interference are facilitated by reducing the separation between the sources of light producing interference. Further, in the Young's double slit experiment the distance between two sources and the screen should be large. The contrast between the bright and dark fringes is improved by making equal the amplitudes of the light sources producing interference. Further, the sources must be narrow and monochromatic. The concept of coherence is discussed in greater detail in the chapter on lasers.

1.5 TYPES OF INTERFERENCE

The phenomenon of interference is divided into two classes depending on the mode of production of interference. These are (*a*) interference produced by the division of wavefront and (*b*) interference produced by the division of amplitude. In the first case the incident wavefront is divided into two parts by making use of the phenomenon of reflection, refraction or diffraction. The two parts of the wavefront travel unequal distances and reunite to produce interference fringes. Young's double slit experiment is a classic examples for this. In Young's double slit experiment one uses two narrow slits to isolate beams from separate portions of the primary wavefront. In the second case the amplitude of the incident light is divided into two parts either by parallel reflection or refraction. These light waves with divided amplitude reinforce after travelling different distances and produce interference. Newton's rings is an example for this type.

1.6 INTERFERENCE IN THIN FILMS

The colours of thin films, soap bubbles and oil slicks can be explained as due to the phenomena of interference. In all these examples, the formation of interference pattern is by the division

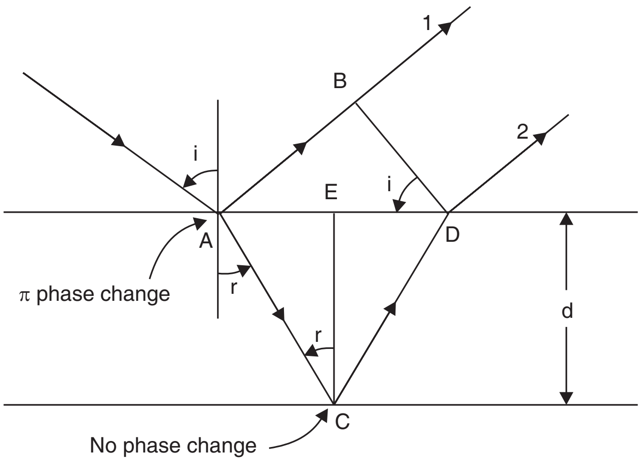

Fig. 1.8 Interference in plane parallel films (Reflection geometry)

of amplitude. For example, if a plane wave falls on a thin film then the wave reflected from the upper surface interferes with the wave reflected from the lower surface. Such studies have many practical applications as provided by the example of production of non-reflecting coatings.

1.6.1 Interference in Plane Parallel Films due to Reflected Light

Let us consider a plane parallel film as shown in the Fig. 1.8. Let light be incident at A. Part of the light is reflected toward B and the other part is refracted into the film towards C. This second part is reflected at C and emerges at D, and is parallel to the first part. At normal incidence, the path difference between rays 1 and 2 is twice the optical thickness of the film.

$$\Gamma = 2\mu d$$

At oblique incidence the path difference is given by

$$\Gamma = \mu(AC + CD) - AB = \frac{2\mu d}{\cos r} - AB$$

$$= \frac{2\mu d}{\cos r} - 2\mu d \tan r \sin r \qquad [\because\ AB = AD \sin i = 2AE.\ \sin i = 2d \tan r.\ \sin i = 2d \tan r.\ \mu \sin r]$$

i.e.,
$$\Gamma = 2\mu d\left\{\frac{1}{\cos r} - \tan r \sin r\right\} = 2\mu d\left\{\frac{1-\sin^2 r}{\cos r}\right\} = 2\mu d.\cos r$$

where μ is the refractive index of the medium between the surfaces. Since for air μ = 1, the path difference between rays 1 and 2 is given by

$$\Gamma = 2d \cos r$$

While calculating the path difference, the phase change that might occur during reflection has to be taken into account. *Whenever light is reflected from an interface beyond which the medium has lower index of refraction, the reflected wave undergoes no phase change. When the medium beyond the interface has a higher refractive index there is phase change of π. The transmitted waves do not experience any phase change.*

Hence, the condition for maxima for the air film to appear bright is

$$2\mu d \cos r + \frac{\lambda}{2} = n\lambda$$

or
$$2\mu d \cos r = n\lambda - \frac{\lambda}{2}$$
$$= (2n-1)\frac{\lambda}{2} \qquad \text{where } n = 1, 2, 3, \ldots$$

The film will appear dark in the reflected light when

$$2\mu d \cos r + \frac{\lambda}{2} = (2n+1)\frac{\lambda}{2}$$

or
$$2\mu d \cos r = n\lambda \qquad \text{where } n = 0, 1, 2,3 \ldots$$

1.6.2 Interference in Plane Parallel Films due to Transmitted Light

Figure 1.9 illustrates the geometry for observing interference in plane parallel films due to transmitted light. We have two transmitted rays *CT* and *EU* which are derived from the same point source and hence, are in a position to interfere. The effective path difference between these two rays is given by

$$\Gamma = \mu(CD + DE) - CP$$

But
$$\mu = \frac{\sin i}{\sin r} = \frac{CP/CE}{QE/CE} = \frac{CP}{QE} \qquad \Rightarrow \qquad CP = \mu(QE)$$

or
$$\Gamma = \mu(CD + DQ + QE) - \mu(QE)$$
$$= \mu(CD + DQ) = \mu(ID + DQ) = \mu(QI)$$
$$= 2\mu d \cos r$$

In this case it should be noted that, no phase change occurs when the rays are refracted unlike in the case of reflection. Hence, the condition for maxima is $2\mu d \cos r = n\lambda$ and the condition for minima is $2\mu d \cos r = (2n-1)\,\dfrac{\lambda}{2}$.

Thus, the conditions of maxima and minima in transmitted light are just the reverse of the condition for reflected light.

1.6.3 Interference in Wedge Shaped Film

Let us consider two plane surfaces *GH* and G_1H_1 inclined at an angle α and enclosing a wedge shaped film (Fig. 1.10). The thickness of the film increases from *G* to *H* as shown in the figure. Let μ be the refractive index of the material of the film. When this film is illuminated there is

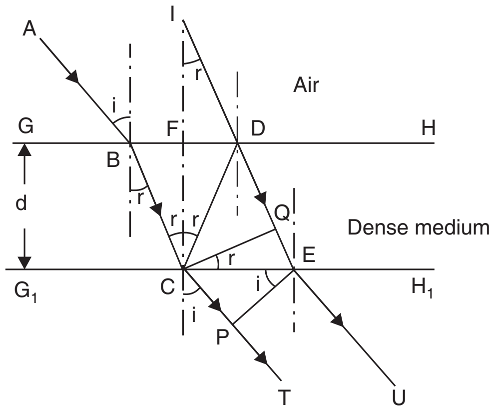

Fig. 1.9 Interference in plane parallel films (Transmission geometry)

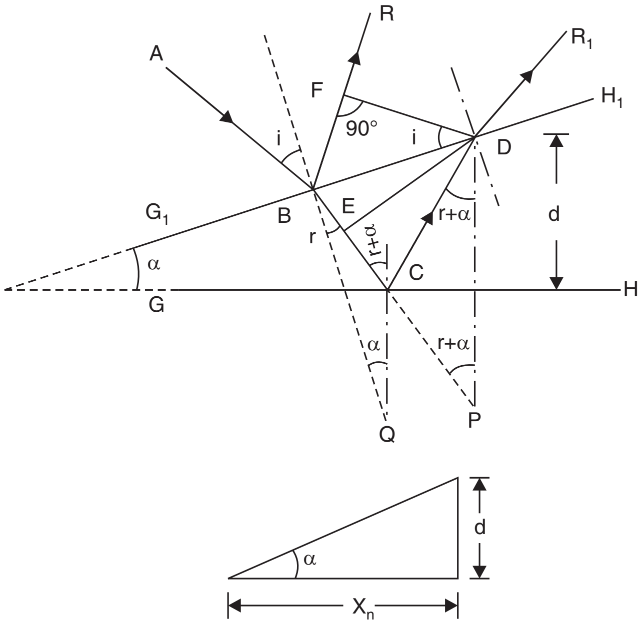

Fig. 1.10 Interference in a wedge shaped film

interference between two systems of rays, one reflected from the front surface and the other obtained by internal reflection at the back surface.

The path difference Γ is given by

$$\Gamma = \mu(BC + CD) - BF$$

$$\Gamma = \mu(BE + EC + CD) - \mu BE$$

$$\left[\because \sin i = \frac{BF}{BD}; \sin r = \frac{BE}{BD};\ \mu = \frac{\sin i}{\sin r} \quad \Rightarrow \quad \mu = \frac{BF}{BE}\right]$$

$$\Gamma = \mu(EC + CD) = \mu(EC + CP) = \mu EP = 2\mu d \cos(r + \alpha)$$

Due to reflection an additional phase difference of λ/2 is introduced.

Hence, $\Gamma = 2\mu d \cos (r + \alpha) + \lambda/2$

For constructive interference

$$2\mu d \cos (r + \alpha) + \lambda/2 = n\lambda$$

or $$2\mu d \cos (r + \alpha) = (2n - 1)\, \lambda/2 \qquad \text{where } n = 1, 2, 3 \ldots$$

For destructive interference

$$\therefore \quad 2\mu d \cos (r + \alpha) + \frac{\lambda}{2} = (2n + 1)\frac{\lambda}{2}$$

or $$2\mu d \cos (r + \alpha) = n\lambda \qquad \text{where } n = 0, 1, 2, 3 \ldots$$

Spacing between two consecutive bright bands is obtained as follows.

For n^{th} maxima

$$2\mu d \cos (r + \alpha) = (2n - 1)\frac{\lambda}{2}$$

Let this band be obtained at a distance X_n from thin edge as shown in Fig. (1.10). For near normal incidence, $r = 0$. Assuming, $\mu = 1$,

From the figure, $d = X_n \tan \alpha$

$$\therefore \quad 2X_n \tan \alpha \cos \alpha = (2n - 1)\frac{\lambda}{2}$$

$$2X_n \sin \alpha = (2n - 1)\frac{\lambda}{2}$$

For $(n + 1)^{\text{th}}$ maxima

$$2X_{n+1} \sin \alpha = (2n + 1)\frac{\lambda}{2}$$

$$\therefore \quad 2(X_{n+1} - X_n) \sin \alpha = \lambda$$

or fringe spacing, $$\beta = X_{n+1} - X_n = \frac{\lambda}{2 \sin \alpha} = \frac{\lambda}{2\alpha}$$

where α is small and measured in radians.

1.7 COLOURS OF THIN FILMS

The discussion of the interference due to a parallel film and at a wedge should now enable us to understand as to why films appear coloured. To summarize, the incident light is split up by reflection at the top and bottom of the film. The split rays are in a position to interfere and interference of these rays is responsible for colours. Since the interference condition is a function of thickness of the film, the wavelength and the angle of refraction, different colours are observed at different positions of the eye. The colours for which the condition of maxima will be satisfied will be seen and others will be absent. It should be noted here that the conditions for maxima and minima in transmitted light are opposite to that of reflected light. Hence, the colours that are absent in reflected light will be present in transmitted light. The colours observed in transmitted and reflected light are complimentary.

1.8 NEWTON'S RINGS

When a plano-convex lens with its convex surface is placed on a plane glass plate, an air film of gradually increasing thickness is formed between the two. If monochromatic light is allowed to fall normally and viewed as shown in the Fig. 1.11 then alternate dark and bright circular fringes are observed. The fringes are circular because the air film has a circular symmetry. Newton's rings are formed because of the interference between the waves reflected from the top and bottom surfaces of the air film formed between the plates as shown in the Fig. 1.12.

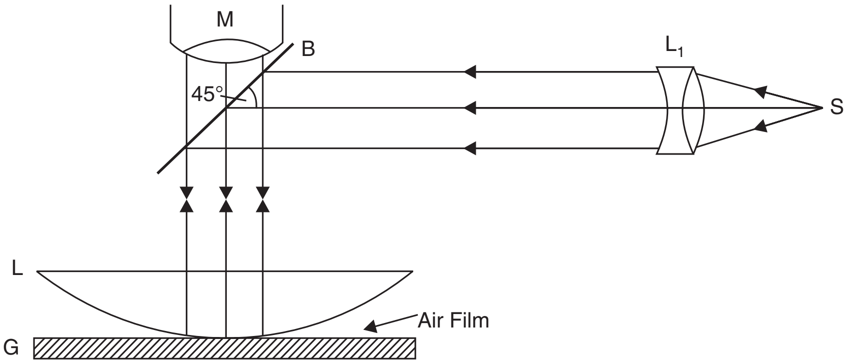

Fig. 1.11 Experimental set up for viewing Newton's rings

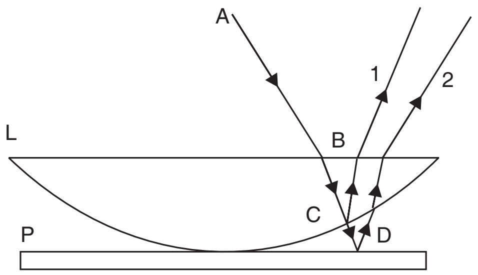

Fig. 1.12 Interference in Newton's rings setup

The path difference Γ between these rays (*i.e.,* rays 1 and 2) is

$$2\mu d \cos r + \frac{\lambda}{2}$$

i.e., Since $r \approx 0$, $\mu = 1$; $\Gamma = 2d + \frac{\lambda}{2}$

At the point of contact $d = 0$, the path difference is $\frac{\lambda}{2}$. Hence, the central spot is dark.

The condition for bright fringe is

$$2d + \frac{\lambda}{2} = n\lambda \quad \text{or} \quad 2d = \frac{(2n-1)\lambda}{2}, \quad \text{where } n = 1, 2, 3 \ldots$$

and the condition for dark fringe is

$$2d + \frac{\lambda}{2} = (2n+1)\frac{\lambda}{2} \quad \text{or} \quad 2d = n\lambda \quad \text{where } n = 0, 1, 2, 3 \ldots$$

Now let us calculate the diameters of these fringes. Let *LOL′* be the lens placed on the glass plate *AB* (Fig. 1.13). The curved surface *LOL′* is part of the spherical surface with the centre at *C*. Let *R* be the radius of curvature and *r* be the radius of Newton's ring corresponding to constant film thickness *d*.

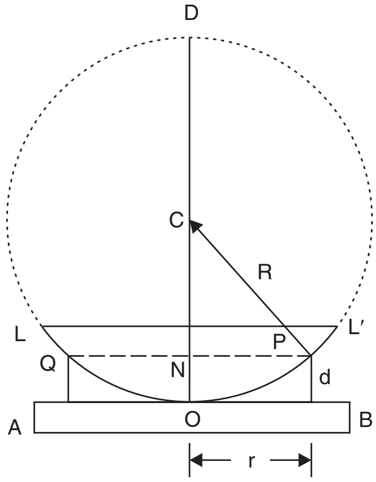

Fig. 1.13 Calculation of diameter of Newton's Ring

From the property of the circle.

i.e., $$NP \times NQ = NO \times ND$$

i.e., $$r \times r = d(2R - d) = 2Rd - d^2 \approx 2Rd$$

i.e., $$r^2 = 2Rd \quad \text{or} \quad d = r^2/2R$$

Thus, for a bright fringe

$$\frac{2r^2}{2R} = \frac{(2n-1)\lambda}{2} \quad \text{or} \quad r^2 = \frac{(2n-1)\lambda R}{2}$$

Replacing r by $D/2$ where D is the diameter we get

$$D_n = \sqrt{2\lambda R}\,\sqrt{2n-1}$$

Similarly, for a dark fringe

$$\frac{2r^2}{2R} = n\lambda \quad \text{or} \quad r^2 = n\lambda R$$

$$D_n^2 = 4n\lambda R$$

$$D_n = 2\sqrt{n\lambda R}$$

Thus, the diameters of the rings are proportional to the square roots of the natural numbers.

By measuring the diameter of the Newton's rings, it is possible to calculate the wavelength of light as follows. We have for the diameter of the n^{th} dark fringe.

$$D_n^2 = 4n\lambda R$$

Similarly diameter for the $(n + p)^{th}$ dark fringe

$$D^2_{n+p} = 4(n + p)\lambda R$$

$\therefore$ $$D^2_{n+p} - D_n^2 = 4\,pR\,\lambda$$

or $$\lambda = \frac{D^2_{n+p} - D_n^2}{4pR}$$

λ can be calculated using this formula.

Newton's rings set up could also be used to determine the refractive index of a liquid. First the experiment is performed when there is air film between the lens and the glass plate. The diameters of the n^{th} and $(n + p)^{th}$ fringes are determined. Then we have

$$D^2_{n+p} - D^2_n = 4p\lambda R$$

Now the liquid whose refractive index is to be determined is poured into the container without disturbing the entire arrangement. Again the diameter of the n^{th} and $(n + p)^{th}$ dark fringes are determined. Again we have

$$D'^2_{n+p} - D'^2_n = \frac{4p\lambda R}{\mu}$$

from the above equations

$$\mu = \frac{D^2_{n+p} - D_n^2}{D'^2_{n+p} - D'^2_n}.$$

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SOLVED EXAMPLES

1. Two narrow and parallel slits 0.08 cm apart are illuminated by light of frequency 8×10^{11} kHz. It is desired to have a fringe width of 6×10^{-4} m. Where should the screen be placed from the slits?

   Solution:

   $$d = 0.08 \text{ cm} = 0.08 \times 10^{-2} \text{ m}, \beta = 6 \times 10^{-4} \text{ m}$$

   frequency $\nu = 8 \times 10^{11}$ kHz

   i.e., $$\lambda = \frac{c}{v} = \frac{3 \times 10^8}{8 \times 10^{11} \times 10^3} \text{ m}, D = ?$$

   From $$\beta = \frac{\lambda D}{d} \text{ we have } D = \frac{\beta d}{\lambda}$$

   $$\therefore \quad D = \frac{6 \times 10^{-4} \times 0.08 \times 10^{-2} \times 8 \times 10^{14}}{3 \times 10^8} = 1.28 \text{ m}.$$

2. In Young's double slit experiment, a source of light of wavelength 4200 Å is used to obtain interference fringes of width 0.64×10^{-2} m. What should be the wavelength of the light source to obtain fringes 0.46×10^{-2} m wide, if the distance between screen and the slits is reduced to half the initial value?

Solution:

In the first case $\lambda = 4200 \text{ Å} = 4200 \times 10^{-10}$ m

$\beta = 0.64 \times 10^{-2}$ m

$$\therefore \quad 0.64 \times 10^{-2} = \frac{4200 \times 10^{-10} \times D}{d} \qquad (i)$$

In the second case $\beta = 0.46 \times 10^{-2}$ m, $\lambda = ?$

$$0.46 \times 10^{-2} = \frac{\lambda \times D/2}{d} = \frac{\lambda D}{2d} \qquad (ii)$$

Dividing equation (*i*) by (*ii*)

$$\frac{0.64 \times 10^{-2}}{0.46 \times 10^{-2}} = \frac{4200 \times 10^{-10} \times D}{d} \times \frac{2d}{\lambda d}$$

$$\therefore \quad \lambda = \frac{4200 \times 10^{-10} \times 2 \times 0.46}{0.64} = 6037.5 \text{ Å}.$$

3. In Young's double slit experiment, the distance between the slits is 1 mm. The distance between the slit and the screen is 1 meter. The wavelength used in 5893 Å. Compare the intensity at a point distance 1 mm from the centre to that at its centre. Also find the minimum distance from the centre of a point where the intensity is half of that at at the centre.

Solution:

Path difference at a point on the screen distance y from the central point

$$= \frac{Y . d}{D}$$

Here $Y = 1 \text{ mm} = 1 \times 10^{-3}$ m

$D = 1$m

$d = 1 \text{ mm} = 1 \times 10^{-3}$ m

$$\therefore \quad \text{Path difference} = \frac{1 \times 10^{-3} \times 1 \times 10^{-3}}{1} = 1 \times 10^{-6} \text{ m} = \Delta$$

$$\text{Phase difference} = \frac{2\pi}{\lambda}\Delta = \frac{10^{-6} \times 2 \times \pi}{5893 \times 10^{-10}} = 3.394\,\pi \text{ radians}$$

$\therefore$ Ratio of intensity with the central maximum

$$= \cos^2 \delta/2 = \cos^2 (1.697\pi) = 0.3372$$

When the intensity is half of the maximum, if δ is the phase difference, we have

$$\cos^2 \delta/2 = 0.5 \quad \text{or} \quad \delta/2 = 45° \quad \text{or} \quad \delta = 90° = \pi/2$$

$$\text{Path difference} = \Delta = \delta\frac{\lambda}{2\pi} = \frac{\pi}{2} \times \frac{\lambda}{2\pi} = \frac{\lambda}{4}$$

Distance of the point on the screen from the centre $= Y = \Delta . \dfrac{D}{d}$

$$= \frac{\lambda}{4} \times \frac{1}{1 \times 10^{-3}} = \frac{5893 \times 10^{-10}}{4 \times 10^{-3}} = 1.473 \times 10^{-4} \text{ m}.$$

4. In a double slit experiment, fringes are produced using light of wavelength 4800 Å. One slit is covered by a thin plate of glass of refractive index 1.4 and the other slit by another plate of glass of the same thickness but of refractive index 1.7. On doing so the central bright fringe shifts to the position originally occupied by the fifth bright fringe from the centre. Find the thickness of the glass plate.

Solution:

We have $n\lambda = (\mu - \mu')t$

Here $n = 5$

$\mu - \mu' = 0.3$

$\lambda = 4800 \times 10^{-10}$ m

$$\therefore \quad t = \frac{5 \times 4800 \times 10^{-10}}{0.3} = 8.0 \times 10^{-8} \text{ m}.$$

5. A drop of oil of volume 0.2 cc is dropped on a surface of tank of water of area 1 m^2. The film spreads uniformly over the whole surface. White light which is incident normally is observed through a spectrometer. The spectrum is seen to contain one dark band whose centre has a wavelength 5.5×10^{-5} cm in air. Find the refractive index of oil.

Solution:

The thickness of the film $= d = \dfrac{0.2 \text{ cm}}{100 \times 100} = 2 \times 10^{-5}$ cm

The film appears dark by reflected light for a wavelength λ given by the relation $2\mu d \cos r = n\lambda$

For normal incidence $r = 0$, $\cos r = 1$

Further $n = 1$ and $\lambda = 5.5 \times 10^{-5}$ cm

$$\mu = \frac{n\lambda}{2t \cos r} = \frac{1 \times 5.5 \times 10^{-5}}{2 \times 2 \times 0^{-5} \times 1} = 1.375.$$

6. A soap film 5×10^{-5} cm thick is viewed at an angle of 35° to the normal. Find the wavelengths of light in the visible spectrum which will be absent from the reflected light ($\mu = 1.33$).

Solution:

Let i be the angle of incidence and r the angle of refraction.

Then $\mu = \dfrac{\sin i}{\sin r}$; $\quad 1.33 = \dfrac{\sin 35°}{\sin r}$

$\Rightarrow \quad r = 25.55° \quad \cos r = 0.90$

Applying the relation, $2\mu d \cos r = n\lambda$

where $d = 5 \times 10^{-5}$ cm

(*i*) For the first order $n = 1$

$$\lambda_1 = 2 \times 1.33 \times 5 \times 10^{-5} \times 0.90 = 12.0 \times 10^{-5} \text{ cm}$$

Which lies in the infrared (invisible) region.

(*ii*) For the second order $n = 2$

$$\lambda_2 = 1.33 \times 5 \times 10^{-5} \times 0.90 = 6.0 \times 10^{-5} \text{ cm}$$

which lies in the visible region.

(*iii*) Similarly, taking $n = 3$, $\lambda_3 = 4.0 \times 10^{-5}$ cm which also lies in the visible region.

(*iv*) If $n = 4$, $\lambda_4 = 3.0 \times 10^{-5}$ cm

which lies in the ultraviolet (invisible region). Hence, absent wavelengths in the reflected light are 6.0×10^{-5} and 4.0×10^{-5} cm.

7. Two glass plates enclose a wedge shaped air film, touching at one edge and separated by a wire of 0.05 mm diameter at a distance 15 cm from that edge. Calculate the fringe width. Monochromatic light of $\lambda = 6000$ Å from a broad source falls normally on the film.

Solution:

Fringe width $$\beta = \frac{\lambda}{2\alpha}$$

Clearly $$\alpha = \frac{0.05 \text{ mm}}{15 \text{ cm}} = \frac{0.005}{15} \text{ radian}$$

$$\beta = \frac{\lambda}{2\alpha} = \frac{6000 \times 10^{-9} \times 15}{2 \times 0.005} = 0.09 \text{ cm.}$$

8. An air wedge of angle 0.01 radians is illuminated by monochromatic light of 6000 Å falling normally on it. At what distance from the edge of the wedge, will the 10th fringe be observed by reflected light.

Solution:

Here $\alpha = 0.01$ radians $n = 10$

$$\lambda = 6000 \times 10^{-10} \text{ m}$$

$$2d = \frac{(2n-1)\lambda}{2}$$

where d is the thickness of wedge

But $$\alpha = \frac{a}{x}$$

$$d = \alpha\, x$$

$\therefore$ $$2\alpha\, x = \frac{(2n-1)\lambda}{2}$$

Here $n = 10$

$$x = \frac{(2n-1)\lambda}{4\alpha} = \frac{19 \times 6000 \times 10^{-10}}{4 \times 0.01} \text{ m} = 2.85 \times 10^{-4} \text{ m.}$$

9. A thin equiconvex lens of focal length 4 meters and refractive index 1.50 rests on and is in contact with an optical flat. Using light of wavelength 5460 Å, Newton's rings are viewed normally by reflection. What is the diameter of the 5th bright ring?

Solution:

The diameter of the nth bright ring is given by

$$D_n = \sqrt{2(2n-1)\lambda R}$$

Here $n = 5$ $\lambda = 5460 \times 10^{-6}$ cm

$$f = 400 \text{ cm} \qquad \mu = 1.50$$

We have

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

Here $R_1 = R_2 = R$

$$\therefore \qquad \frac{1}{f} = (\mu - 1)\frac{2}{R}$$

i.e., $$\frac{1}{400} = (1.50 - 1)\frac{2}{R} \quad \Rightarrow \quad R = 400 \text{ cm}$$

$$\therefore \qquad D_n = \sqrt{2 \times (2 \times 5 - 1) \times 5460 \times 10^{-6} \times 400} = 0.627 \text{ cm.}$$

10. In Newton's ring experiment, the diameters of the 4th and 12th dark rings are 0.400 cm and 0.700 cm respectively. Find the diameter of the 20th dark ring.

Solution:

We have $D^2_{n+p} - D^2_n = 4p\lambda R$

Here $(n + p) = 12,\ n = 4,\ p = 12 - 4 = 8$

$$\therefore \qquad D_{12}^2 - d_4^2 = 4 \times 3 \times \lambda R \ ... \qquad (i)$$

$$D_{20}^2 - D_4^2 = 4 \times 16 \times \lambda R \ ... \qquad (ii)$$

Dividing (*ii*) by (*i*)

$$\frac{D_{20}^2 - D_4^2}{D_{12}^2 - D_4^2} = \frac{4 \times 16 \times \lambda R}{4 \times 8 \times \lambda R} = 2$$

$$\frac{D_{20}^2 - (0.4)^2}{(0.7)^2 - (0.4)^2} = 2 \quad \Rightarrow \quad D_{20} = 0.906 \text{ cm.}$$

11. In a Newton's ring experiment the diameter of the 10th ring changes from 1.40 to 1.27 cm when a liquid is introduced between the lens and the plate. Calculate the refractive index of the liquid.

Solution:

When the liquid is used the diameter of the 10th ring is given by

$$(D'_{10})^2 = \frac{4 \times 10 \times \lambda R}{\mu} \qquad (i)$$

For air medium

$$(D_{10})^2 = 4 \times 10 \times \lambda R \qquad (ii)$$

Dividing (*i*) by (*ii*)

$$\mu = \frac{D_{10}^2}{D'^2_{10}} = 1.215.$$

12. In a Newton's ring experiment the diameter of the 5th dark ring was 0.3 cm and the diameter of the 25th ring was 0.8 cm. If the radius of the curvature of the plano-convex lens is 100 cms, find the wavelength of the light used.

Solution:

$$\lambda = \frac{D_{n+p}^2 - D_n^2}{4pR}$$

Here $D_{25} = 0.8$ cm $\quad D_5 = 0.3$ cm

$P = 25 - 5 = 20$ and $R = 100$ cm

$$\therefore \quad \lambda = \frac{(0.8)^2 - (0.3)^2}{4 \times 20 \times 100} = 4.87 \times 10^{-5} \text{ cm.}$$

QUESTIONS

1. What is interference of light waves? What are the conditions necessary for obtaining interference fringes?
2. Two independent non-coherent sources of light cannot produce an interference pattern. Why?
3. Define spatial and temporal coherence.
4. Describe Young's double slit experiment and obtain an expression for fringe width.
5. Write a note on colours of thin films.
6. Show that colours exhibited by reflected and transmitted systems are complementary.
7. Find an expression for the width of the fringes obtained in the case of air wedge. How would you use the result to find the wavelength of a given monochromatic radiation?
8. What are Newton's rings? How are they formed? Why are they circular?
9. Explain why the centre of Newton's rings is dark in the reflected system.
10. Describe how you would use Newton's rings to determine the wavelength of a monochromatic radiation and derive the relevant formula.
11. Obtain an expression for the radius of the n^{th} dark ring in the case of Newton's rings.
12. Show that the radii of Newton's rings are in the ratio of the square roots of the natural numbers.

PROBLEMS

1. Interference fringes are formed on a screen which is at a distance of 0.8 m. It is found that the fourth bright fringe is situated at a distance of 0.00108 m from the central fringe. Calculate the distance between the two coherent sources. (given λ = 5896 Å). (**Ans.** 1.75×10^{-19} m)
2. A parallel beam of light ($\lambda = 5890 \times 10^{-10}$ m) is incident on a thin glass plate ($\mu = 1.5$) such that the angle of refraction into the plate is 60°. Calculate the smallest thickness of plate which would appear dark by reflection. (**Ans.** 3.926×10^{-7} m)
3. White light falls normally on a film of soapy water whose thickness is 5×10^{-5} cm and $\mu = 1.33$. Which wavelength in the visible region will be reflected most strongly? (**Ans.** 5320×10^{-10} m)
4. White light is incident on a soap film at an angle of $\sin^{-4} 4/5$ and the reflected light on examination by a spectroscope shows dark bands. Two consecutive bands corespond to wavelength 6.1×10^{-5} and 6.0×10^{-5} cm. If $\mu = 4/3$, calculate its thickness. (**Ans.** 1.7×10^{-5} m)
5. If the angle of the air wedge is 0.25° and the wavelengths of sodium lines are 5896 and 5890 Å, find the distance from the apex at which the maximum due to two wavelengths first coincide when observed in reflected light. (**Ans.** 6.63 cm)
6. A monochromatic light of wavelength 5893×10^{-10} m falls normally on an air wedge. If the length of the wedge is 0.05 m, calculate the distance at which the 12th dark and 12th bright fringes will form the line of contact of the glass plates forming the wedge. (Given the thickness of the specimen = 154×10^{-6} m). (**Ans.** 9.61×10^{-4} m, 9.21×10^{-4} m)

7. A square piece of cellophane film with refractive index 1.5 has a wedge shaped section so that its thickness at two opposite sides is t_1 and t_2. If with a light of λ = 6000 Å, the number of fringes appearing in the film is 10, calculate the difference $t_2 - t_1$. (**Ans.** 2×10^{-4} cm)
8. A Newton's ring arrangement is used with a source emitting two wavelengths $\lambda_1 = 6 \times 10^{-5}$ m and $\lambda_2 = 4.5 \times 10^{-5}$ m. It is found that nth dark ring due to λ_1 coincides with $(n + 1)$th dark ring for λ_2. If radius of curvature of the lens is 90 cm find the diameter of the nth dark ring. (**Ans.** 0.254 cm)
9. Light containing two wavelengths λ_1 and λ_2 falls normally on a planoconvex lens of radius of curvature R resting on a glass plate. If the nth dark ring due to λ_1, coincides with $(n = 1)$th dark ring due to λ_2, prove that the radius of the nth dark ring of λ_1 is $\sqrt{\dfrac{\lambda_1\lambda_2 R}{(\lambda_1 - \lambda_2)}}$.
10. Newton's rings formed by sodium light between a flat glass plate and a convex lens are viewed normally. What will be the order of the dark ring which will have double the diameter of 40th ring? (**Ans.** 160)